Exploring Einstein's

Special Relativity

Seminar Outline

The Cast

Sir Isaac Newton (1643-1727, British) is most famous for his laws of physics, and especially his equation **F=ma**. Newton quantified Kepler's laws, which were statistically-based, into mathematical discipline. Newtonian physics is still used today in classrooms and are quite valid for non-relativistic, or classic, speeds (where movement is a small percentage of the speed of light). However, for advanced applications (e.g. GPS), Newton's laws are not adequate. "Classic Physics" is a synonym for "Newtonian Physics".

James Clerk Maxwell (1831-1879, Scotsman) unified electricity and magnetism (1864) by realizing that they were two sides of the same coin, joining these two forces into what's now called the "electromagnetic force". Maxwell calculated the speed of electromagnetic fields before anyone was sure that light was an electromagnetic wave. All light (electromagnetic waves) of whatever frequency, obey Maxwell's four elegant laws.

Albert Einstein (1879-1955, German) is most remembered for his equation $\mathbf{E=mc^2}$ which, a la Maxwell, unified mass and energy – but his contributions went way beyond that. In 1905, he published his "Special Theory of Relativity" (the topic of this session), which reconciled differences between Newton and Maxwell, and whose calculations are still used today. Special Relativity deals with for free-moving reference-frames, where all forces (including gravity) are absent. Einstein's "General Theory of Relativity" (1915) included gravity in the relativity equations, and these are still used verbatim today. In terms of physics, "relativistic" is a synonym for "at a significant percentage of the speed of light" at which point Newton ("classic") starts becoming somewhat wrong and Einstein starts becoming more right.

Fundamental Assumptions

- ***"The laws of physics are the same for <u>all observers in uniform motion</u> relative to each other."*** This statement is consistent for Newtonian (classical) physics, but contradicts Maxwell's principles of electromagnetism.

- ***"The speed of light in a vacuum is <u>the same for all observers</u>, regardless of their relative motion or of the motion of the source of the light."*** In other words, any attempt to measure the speed of light will produce the same result (~186,000 miles/sec) regardless of whether the observer is in motion relative to the light, or the light is in motion relative to the observer, or both. This is consistent with Maxwell, but inconsistent with Newton.

Consequences:

- **Lack of Simultaneity:** If two observers are in uniform motion, two events which appear simultaneous to one observer may not be simultaneous to the other.

- **Time Dilation:** If one observer is in motion with respect to a second observer (at rest), the second observer will observe a clock in the first observer's reference frame running more slowly compared to the second observer's clock.

- **Length Contraction:** Objects moving in a direction will contract in that direction.

- **There is more…** Not covered in this seminar: mass dilation, summing of velocities, etc.

Lack of Simultaneity

We start with a train, with observer O1 in the exact middle of the train, and a fixed platform, with observer O2 in the exact center of the platform. The train moves left to right.

```
   ___Train__
  |          |-->
  |____O1____|
    A___________B
     |    O2    |
     |__________|
       Platform
```

When the train is exactly aligned with the platform – that is, when O1 and O2 are directly opposite each other – these things happen at points (“events”) A and B:

- Smudges are painted onto both the train and the platform
- A light flashes at both these points.
- The light proceeds to both observers O1 and O2.

```
       ___Train__
      |          |-->
      |____O1____|
    A___________B
     |    O2    |
     |__________|
       Platform
```

At some point as the train moves forward:

- The light from event B reaches O1, but has not yet reached O2.
- The light from event A has reached neither observer.
- Thus, observer O1 sees event B, but not yet event A. Observer O2 has not yet seen neither flash.

```
         ___Train__
        |          |-->
        |____O1____|
    A___________B
     |    O2    |
     |__________|
       Platform
```

- Somewhat farther on, the lights from both events A and B reach observer O2, who sees these events as simultaneous.

```
     ___Train__
    |          |-->
    |____O1____|
  A__________B
  |    O2    |
  |__________|
    Platform
```

- Even farther down the track, the light from event A finally reaches observer O1.

```
      ___Train__
     |          |-->
     |____O1____|
  A__________B
  |    O2    |
  |__________|
    Platform
```

Thus, observer O2 sees events A and B as being simultaneous, but observer O1 disagrees, stating that event B happened first.

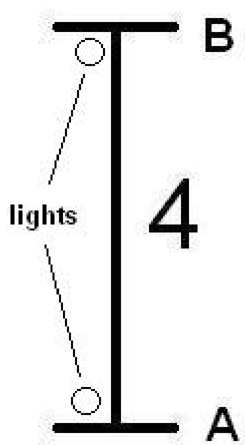
Time Dilation

"Moving Clocks Run Slow"

The Players

- The **Traveler –** me – has a light clock, as described below. The Traveler will be "in motion" with respect to the Observer.
- The **Observer –** you – is "at rest".

The Traveler's Clock

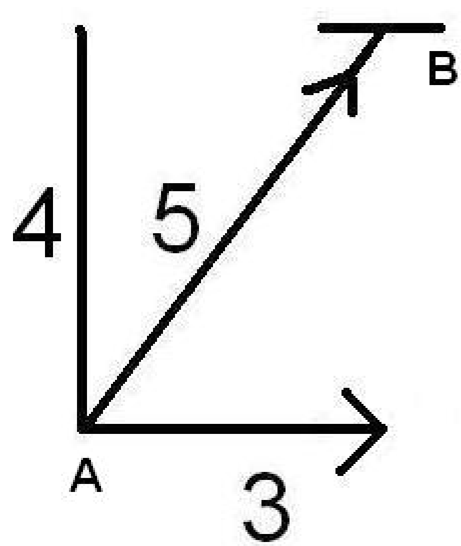


The Traveler's Clock is a light-clock. When the Traveler pushes a button, the following things happen:

- The bottom light flashes, and at the same time, a pulse of light is sent from the bottom emitter toward the top collector. When this happens, the Traveler starts his stopwatch.
- When the light reaches the top emitter, the top light flashes. The Traveler stops his stopwatch, and it measures 1 second.
- For these thought-experiments, we're going to assume the top and bottom signal lights are "magic" lights that travel at an infinite speed. That is, both the observer and traveler will see them at exactly the same time.

The light-clock is built such that light requires one second to travel 4 distance 'units', from the standpoint of the Traveler at rest. In other words, our "speed of light" is 4 units/second. (We don't measure the Traveler's clock since the Traveler already knows it takes 1 sec for the pulse to travel from bottom to top.)

The Traveler in Motion



Now we put the Traveler in motion so that he goes by the Observer (you), left to right. When the Observer sees the bottom light flash, he starts his stopwatch, and when the top light flashes, he stops the stop watch.

Now, the path of the light, as seen by the Observer, isn't straight up and down; it also has a horizontal component because of the left-right motion. Instead of light traveling 4 distance units (as seen by the Traveler), the Observer sees it traveling 5 units.

Since the speed of light is 4 units/second (for everyone, according to Einstein), and light has traveled 5 units, the elapsed time on the Observer's stopwatch is 1.25 seconds.

NOTE: Our calculations are the same regardless of whether a) the Observer measured the distance as 5 to 4 and used that to calculate elapsed time, or b) measured the time difference (1.25:1) and calculated the distance.

So – for exactly the same event, the Traveler determines the event lasted 1 second, while the Observer determined the elapsed time as 1.25 seconds. From the Observer's standpoint, the Traveler's clock is "running slow".

How Fast Were We Going?

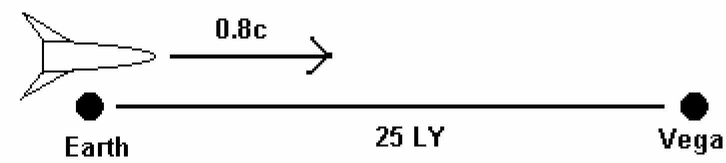
The reason I chose the "4" and "5" in the above example is that my very favorite right-triangle is the 3-4-5 triangle. It keeps the arithmetic from getting messy.

The light-path seen by the Observer is 5 units long, and has a vertical component of 4. Since this is a right-triangle, the horizontal component must be 3. Since light traveled 5 units at the speed of light, and in that time the horizontal motion was 3 units, the Traveler must have been traveling 3/5 = 60% of the speed of light. Indeed, if you look at the Time Dilation Table and find the row for 0.6, you'll see the dilation (t') is 1.25.

Length Contraction

From the Earth to Vega

Vega is a star 25 light-years away, and some astronomers suspect it may have a planet orbiting it, and we want to find out. (A light-year is the distance light travels in a year. At 186,000 miles/second, light travels about 6,000,000,000,000 = 6×10^{12} miles in a year – six trillion miles.)



We don't want to waste time, so we build a spaceship capable of traveling 99.5% of the speed of light (0.995c). Our spaceship can reach Vega in 25 / 0.995 = 25.125 years, as measured by those left behind on Earth.

But, for the folks in the spaceship, time passes more slowly because of Time Dilation ("moving clocks run slow"). Find the **fract[c]** 0.995 entry in the Time Dilation Table, and you'll find the **t'** value is 10.01. This means that when our spaceship reaches Vega, 25.125 years has passed on Earth, but only 25.125 / 10.01 = 2.510 years has passed on the spaceship.

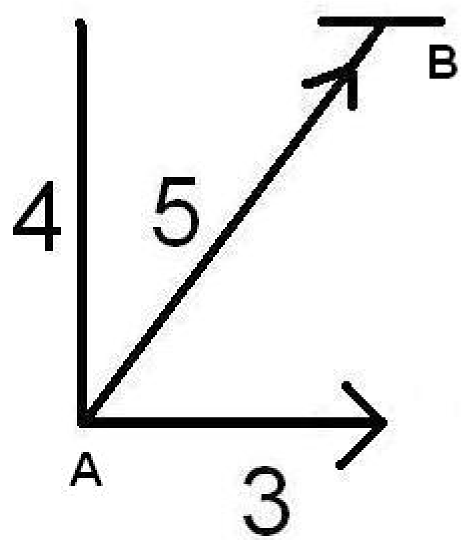
But using Time Dilation alone, this doesn't make sense. If the spaceship's clock shows 2.510 years, and it travels at 0.995 the speed of light, it would have traveled only 2.510 x 0.995 = 2.50 light years – only 10% of the Earth-measured distance.

And it has – at least from the standpoint of the spaceship. Length Contraction says that distances contract (shrink) in the direction of motion. Not only is the spaceship's clock running more slowly than one on Earth, distances are shorter (in the direction of travel) than those measured from Earth.

If you go to the Length Contraction Table, and find the **fract[c]** for 0.995, you'll find the **L'** value 0.100. This means that for speeds of 99.5% of the speed of light, distances (again – in the direction of travel) *to* objects at rest are only 10% of those measured *by* objects also at rest. So, from the standpoint of Earth, you need to travel 25 light-years to reach Vega, but, as measured from the spaceship, it has only traveled 2.5 light-years.

Traveler Revisited

We just saw an example that has both Time Dilation and Length Contraction. But in our Traveler/Observer sample, we just saw Time Dilation. Does that mean that Length Contraction didn't happen in this example?



No – there *was* Length Contraction. Remember where we left off. The Observer (you) noted that the light clock showed 5 distance-units at the speed of light, while the Traveler (me) moved 3 distance units (as measured by you). This allowed us to determine the Traveler's speed to be 3/5 = 60% of the speed of light.

But the Traveler disagrees:

- The Traveler only observes a one-second elapsed time.
- If the distance was 3 after 1.25 seconds, then after only one second, the distance would be 3 x (1/1.25) = 2.4 distance units.
- 2.4 / 3 = 0.8
- If you now go to the Length Contraction table, and go to the 0.6 row, you'll see a Length Contraction value of 0.8 (same as above). Knowing the speed was 0.6 of the speed of light, we could have gone there directly, multiplied 3 by 0.8 = 2.4.

To summarize:

- The reference frame *without* the clock (Observer – you – in our exercise) experiences Time Dilation.
- The reference frame *with* the clock (Traveler – me – experiences Length Contraction.
- Each of these balances the effect of the other.

And to conclude:

- These examples work regardless of which frame is moving and which is at rest. In other words, if the Traveler was at rest and the Observer was moving, the exact same values would be calculated.

The calculations are the same even if *both* reference frames have light-clocks (except there are twice as many of them). If you work this out for yourself, I'd recommend you stay with the 3-4-5 triangle for simplicity of calculation.

Credits

Special thanks go to:

- **Prof. Alex Filippenko**, UC Berkeley, and his Teaching Company DVD series "Understanding the Universe: An Introduction to Astronomy"
- **Prof. Richard Wolfson**, Middlebury College, and his Teaching Company DVD series "Einstein's Relativity and the Quantum Revolution"
- **Prof. Neil deGrasse Tyson**, Director of the Hayden Planetarium, and his Teaching Company DVD series "My Favorite Universe"
- **Wikipedia.com**, for more reference material than can be listed.

Tables

These tables are provided in lieu of equations. They should be close enough for educational work.

Time Dilation

For an object, with a clock, moving relative to you at the fraction of the speed of light **fract[c]**, each unit of time (second, year, etc) that elapses on that clock will result in **t'** units of time on your clock.

fract[c]	t'		fract[c]	T'		fract[c]	t'		fract[c]	t'
0.1	1.01		0.91	2.41		0.991	7.47		0.9991	23.58
0.2	1.02		0.92	2.55		0.992	7.92		0.9992	25.01
0.3	1.05		0.93	2.72		0.993	8.47		0.9993	26.73
0.4	1.09		0.94	2.93		0.994	9.14		0.9994	28.87
0.5	1.15		0.95	3.20		0.995	10.01		0.9995	31.63
0.6	1.25		0.96	3.57		0.996	11.19		0.9996	35.36
0.7	1.40		0.97	4.11		0.997	12.92		0.9997	40.83
0.8	1.67		0.98	5.03		0.998	15.82		0.9998	50.00
0.9	2.29		0.99	7.09		0.999	22.37		0.9999	70.71

Length Contraction

If an object is moving across your field of view at the fraction of the speed of light **fract[c]**, and this object has a rest length of one unit (inches, yards, etc.), the length of this object, as measured by you, would be **L'** units of length.

fract[c]	L'		fract[c]	L'		fract[c]	L'		fract[c]	L'
0.1	0.995		0.91	0.415		0.991	0.134		0.9991	0.042
0.2	0.980		0.92	0.392		0.992	0.126		0.9992	0.040
0.3	0.954		0.93	0.368		0.993	0.118		0.9993	0.037
0.4	0.917		0.94	0.341		0.994	0.109		0.9994	0.035
0.5	0.866		0.95	0.312		0.995	0.100		0.9995	0.032
0.6	0.800		0.96	0.280		0.996	0.089		0.9996	0.028
0.7	0.714		0.97	0.243		0.997	0.077		0.9997	0.024
0.8	0.600		0.98	0.199		0.998	0.063		0.9998	0.020
0.9	0.436		0.99	0.141		0.999	0.045		0.9999	0.014